

## On the self-similarity in quantum Hall systems

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**Abstract.** – The Hall-resistance curve of a two-dimensional electron system in the presence of a strong perpendicular magnetic field is an example of self-similarity. It reveals plateaus at low temperatures and has a fractal structure. We show that this fractal structure emerges naturally in the Hamiltonian formulation of composite fermions. After a set of transformations on the electronic model, we show that the model, which describes interacting composite fermions in a partially filled energy level, is self-similar. This mathematical property allows for the construction of a basis of higher generations of composite fermions. The collective-excitation dispersion of the recently observed 4/11 fractional-quantum-Hall state is discussed within the present formalism.

A branch of a snow flake observed under a microscope appears to be almost the same as the macroscopic flake. This is one of the most prominent examples of self-similarity and fractal structure in nature [1]. Another example of this phenomenon is the curve of the transverse Hall resistance of a two-dimensional (2D) electron system in the presence of a strong perpendicular magnetic field [2]: at low temperatures, the primary sequence of plateaus in the Hall resistance, which occur at specific values of the magnetic field, repeats itself qualitatively in a different field range (see Fig. 1). The phenomenon of plateau formation in the Hall resistance is known as the quantum Hall effect. One distinguishes the integral and the fractional quantum Hall effect (IQHE and FQHE). The IQHE [3] is found at lower magnetic fields  $B$  and is a manifestation of the quantisation of the electron energy, with charge  $-e$ , into equidistant energy levels, called Landau levels (LLs). The level degeneracy is characterised by the flux density  $n_B = eB/h$ , and the filling of the levels is given by  $\nu = n_{el}/n_B$ , where  $n_{el}$  is the electron density. When  $n$  LLs are completely filled ( $\nu = n$ ), one finds the IQHE: when the magnetic field is lowered, the number of states per level is reduced, and some electrons are therefore promoted to a higher LL. These electrons become localised due to residual impurities in the sample and thus do not contribute to the electrical transport. As a consequence, the Hall resistance is not sensitive to the change in the magnetic field and remains at its original quantised value  $R_H = h/e^2n$ . This gives rise to a plateau when the Hall resistance is plotted as a function of  $B$ .

The FQHE is found at fractional values of the filling factor, *i.e.* when a LL - mostly the lowest - is only partially filled [4]. This effect is due to the Coulomb interaction between the electrons,

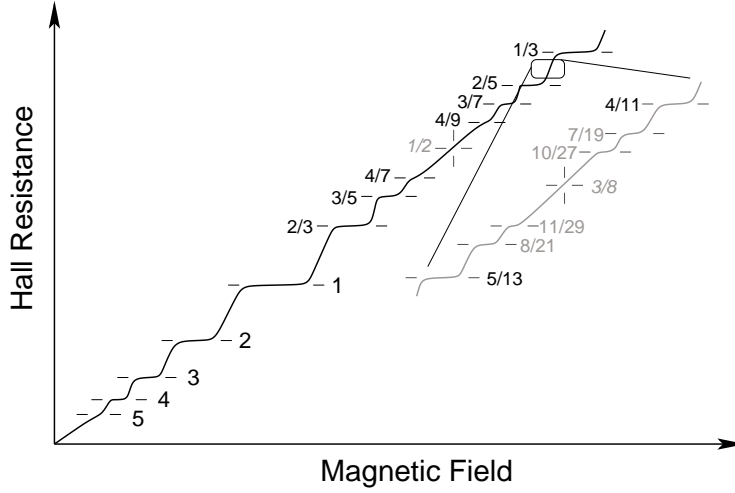


Fig. 1 – Self-similarity of the Hall-resistance curve (c.f. Refs. 2 and 10). The plateaus in the Hall resistance are due to gapped ground states at certain values of the filling factor ( $\nu = n$ , with integral  $n$ , for the IQHE, and  $\nu = p/(2ps + 1)$ , with integral  $s$  and  $p$ , for the FQHE described as an IQHE of CFs). The gray curve on the right is a zoom on the encircled region of the resistance curve, which shows possible states of a second generation of CFs. Only the black states ( $4/11$  and  $5/13$ ) have been observed in the experiments by Pan *et al.* [9].

which lifts the original degeneracy of the LLs. Laughlin explained the existence of gapped ground states at  $\nu = 1/(2s + 1)$  with the help of trial wave functions [5]. Away from these filling factors, fractionally charged quasi-particles or quasi-holes with a finite energy are excited. In analogy with the IQHE, these quasi-particles/-holes become localised, and this leads to the formation of plateaus in the Hall resistance around  $\nu = 1/(2s + 1)$ . The observation of the FQHE at other filling factors such as  $\nu = 2/5, 3/7, 4/9, \dots$ , however, required new concepts beyond Laughlin's original theory [6, 7]. Another scheme was put forward by Jain, who introduced the concept of composite fermions (CFs) [8]: in order to minimise the Coulomb repulsion, an electron binds to a vortex-like excitation of the electron liquid carrying  $2s$  flux quanta. This bound state is exactly the CF [Fig. 2(a)]. Due to their renormalised charge  $e^* = e/(2ps + 1)$ , the CFs experience a reduced coupling to the magnetic field and, in a naive picture, form LLs themselves. The filling  $\nu^*$  of these CF-LLs is related to the electronic filling factor  $\nu$  by

$$\nu = \frac{\nu^*}{2s\nu^* + 1}. \quad (1)$$

The FQHE at  $\nu = p/(2ps + 1)$  may be understood as an IQHE of these CFs when  $p$  CF-LL are completely filled ( $\nu^* = p$ ). This theory classifies the most prominent FQHE states.

Recently, Pan *et al.* have observed a FQHE at  $\nu = 4/11$  [9], which cannot be described in the framework of a theory of non-interacting CFs [8]. It corresponds to a CF filling factor  $\nu^* = 1 + 1/3$  and has therefore been interpreted as a FQHE of CFs. It is natural to generalise the CF picture to higher generations [10–12]: whereas the CFs in the lowest level are treated as inert, CFs in the partially filled first excited CF-LL bind to vortex-like excitations of the CF liquid, and this bound state may be viewed as a CF of the second generation [ $C^2F$ , see Fig. 2(b)]. An IQHE of  $C^2Fs$  and even higher generations would lead to the self-similarity of the Hall-resistance curve, pointed out by Mani and v. Klitzing [2].

Here, we provide a mathematical framework for the understanding of the self-similarity and the fractal structure of the Hall curve based on the Hamiltonian theory of the FQHE, proposed by Murthy and Shankar [13]. The self-similarity of the FQHE is revealed by a reproduction of the original

quantum-mechanical model, which describes the low-energy degrees of freedom, after a set of transformations. Because inter-LL excitations belong to the high-energy part of the spectrum, which is neglected in the model, the kinetic energy is an unimportant constant, and the model of electrons restricted to a single LL is given only by the interaction Hamiltonian

$$\hat{H} = \frac{1}{2} \sum_{\mathbf{q}} v_n(q) \bar{\rho}(-\mathbf{q}) \bar{\rho}(\mathbf{q}), \quad (2)$$

where  $v_n(q) = (2\pi e^2/\epsilon q)[L_n(q^2 l_B^2/2) \exp(-q^2 l_B^2/4)]^2$  is the effective Coulomb repulsion in the  $n$ -th LL, in terms of Laguerre polynomials  $L_n(x)$ . The density operators  $\bar{\rho}(\mathbf{q})$ , projected to a single LL, satisfy the algebra [14]

$$[\bar{\rho}(\mathbf{q}), \bar{\rho}(\mathbf{k})] = 2i \sin\left(\frac{(\mathbf{q} \times \mathbf{k})_z l_B^2}{2}\right) \bar{\rho}(\mathbf{q} + \mathbf{k}). \quad (3)$$

The Hamiltonian (2), together with the commutation relations (3), defines the full quantum mechanical model. The projected density operators  $\bar{\rho}(\mathbf{q})$  may be interpreted as a density of fermions, the degrees of freedom of which are described only by the center of their cyclotron motion  $\mathbf{R}_j^{(e)} = (X_j, Y_j)$ , called the *guiding center* of the  $j$ -th particle,

$$\bar{\rho}(\mathbf{q}) = \sum_j e^{-i\mathbf{q} \cdot \mathbf{R}_j^{(e)}} = \sum_{m, m'} G_{m, m'}(\mathbf{q} l_B) a_m^\dagger a_{m'},$$

where  $a_m^\dagger$  and  $a_m$  are the electron creation and annihilation operators in the partially filled LL, respectively. The matrix elements are given in terms of associated Laguerre polynomials  $L_n^m(x)$  (for  $m \geq m'$ ),

$$G_{m, m'}(\mathbf{z}) = \sqrt{\frac{m'!}{m!}} \left(-\frac{iz}{\sqrt{2}}\right)^{m-m'} L_{m'}^{m-m'}\left(\frac{|z|^2}{2}\right) e^{-|z|^2/4}, \quad (4)$$

with the complex variable  $z = x + iy$ . The unusual commutation relations (3) arise from the non-commutativity of the components of these guiding centers,  $[X_k, Y_l] = -il_B^2 \delta_{k,l}$ , which is precisely the origin of the fact that each quantum-mechanical state occupies a minimal surface  $\sigma = 2\pi l_B^2 = 1/n_B$ , threaded by one flux quantum. The representation of the density operators in the CF basis is obtained in two steps. First, one formally introduces the density of the vortex-like excitations (“pseudo-vortices”), carrying  $2s$  flux quanta,  $\bar{\chi}(\mathbf{q}) = \sum_j \exp(-i\mathbf{q} \cdot \mathbf{R}_j^{(v)})$ , where  $\mathbf{R}_j^{(v)}$  is now the guiding center of the  $j$ -th pseudo-vortex. Because the pseudo-vortex has a charge  $-c^2 = -2ps/(2ps+1)$ , in units of the electron charge [13], its guiding-center components satisfy  $[X_k^{(v)}, Y_l^{(v)}] = i(l_B^2/c^2) \delta_{k,l}$ , and this leads to the same commutation relations for the pseudo-vortex density as for the electron density if one replaces  $l_B^2 \rightarrow -l_B^2/c^2$  in expression (3). This algebraic structure has first been investigated by Pasquier and Haldane, who treated a system of *bosons* at  $\nu = 1$  [15]. The pseudo-vortex artificially introduces new degrees of freedom, which would have to be considered in form of a constraint,  $\bar{\chi}(\mathbf{q})|\psi\rangle = 0$ , for any physical state  $|\psi\rangle$ . It must be taken into account in a conserving approximation if one intends to describe the physical properties of a *compressible* state at  $\nu = 1/2$  [16, 17].

In an approximation, which is valid if the ground state is separated from excited states by an energy gap given by the CF-LL separation [13], one may neglect the constraint and construct the CF density as a superposition of the electronic and pseudo-vortex densities,

$$\bar{\rho}_{CF}(\mathbf{q}) = \bar{\rho}(\mathbf{q}) - c^2 \bar{\chi}(\mathbf{q}).$$

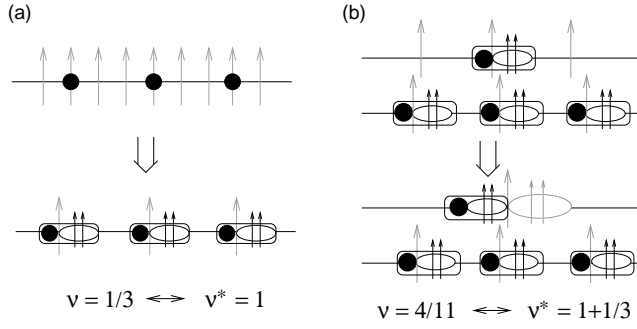


Fig. 2 – Composite fermions. (a) the FQHE at  $\nu = 1/3$  (top panel) may be understood as an IQHE of CFs: each electron (black circle) is bound to a vortex-like excitation (open black ellipses) carrying two flux quanta (small black arrows). One flux quantum per CF remains “free” (big gray arrows) so that the effective CF filling factor is  $\nu^* = 1$ . (b) sketch of the 4/11 state if interpreted in terms of a second generation of CFs at  $\nu^* = 1 + 1/3$ : whereas the CFs carrying two flux quanta in the lowest CF-LL are inert, each CF binds to a pseudovortex (open gray ellipse) with two flux quanta (small gray arrows) so that the total number of flux carried by the  $C^2F$  is four.

In a second step, one performs a variable change to introduce the CF guiding center as a weighted sum of the electron and pseudo-vortex guiding centers [13],

$$\mathbf{R}^{CF} = \frac{\mathbf{R}^{(e)} - c^2 \mathbf{R}^{(v)}}{1 - c^2}, \quad \eta^{CF} = \frac{c}{1 - c^2} (\mathbf{R}^{(e)} - \mathbf{R}^{(v)}). \quad (5)$$

The operator  $\eta^{CF}$  may be interpreted as a CF cyclotron coordinate, responsible for the formation of the CF-LLs. The degeneracy of any CF-LL is obtained from the minimal surface occupied by each state  $\sigma^* = 2\pi l_B^{*2}$  due to the commutation relations of the CF guiding-center components,  $[X^{CF}, Y^{CF}] = -il_B^{*2}$ , where  $l_B^* = l_B / \sqrt{1 - c^2}$  is the CF magnetic length. This yields a reduced flux density  $n_B^* = 1/2\pi l_B^{*2}$ , and one obtains the IQHE of CFs if  $p$  CF-LL are completely filled ( $\nu^* = n_{el}/n_B^* = p$ ).

In analogy with the electronic case, the excitations of lowest energy are those within the same CF-LL  $p$ , and they are only present if this level is partially filled, *i.e.* if  $p < \nu^* < p + 1$ . The CFs in the completely filled lower levels may be considered as inert. Formally this means that one restricts the CF-density operator to the  $p$ -th level,  $\langle \bar{\rho}_{CF}(\mathbf{q}) \rangle_p = F_{CF}^p(q) \bar{\rho}(\mathbf{q})$ , with the projected CF-density operator

$$\bar{\rho}(\mathbf{q}) = \sum_j e^{-i\mathbf{q} \cdot \mathbf{R}_j^{CF}} = \sum_{m, m'} G_{m, m'}(\mathbf{q} l_B^*) b_m^\dagger b_{m'},$$

where  $b_m^\dagger$  and  $b_m$  are the CF creation and annihilation operators in the  $p$ -th CF-LL. The matrix elements, given by Eq. (4), are the same as for the electronic case in terms of the renormalised magnetic length  $l_B^*$ . For the projected CF density operators, one obtains exactly the same algebra as for the projected electron densities (3) if one replaces  $l_B \rightarrow l_B^*$ ,

$$[\bar{\rho}(\mathbf{q}), \bar{\rho}(\mathbf{k})] = 2i \sin \left( \frac{(\mathbf{q} \times \mathbf{k})_z l_B^{*2}}{2} \right) \bar{\rho}(\mathbf{q} + \mathbf{k}). \quad (6)$$

The new model Hamiltonian for restricted CFs is

$$\tilde{H} = \frac{1}{2} \sum_{\mathbf{q}} \tilde{v}(q) \bar{\rho}(-\mathbf{q}) \bar{\rho}(\mathbf{q}), \quad (7)$$

where the CF form factor of the  $p$ -th CF-LL,

$$F_{CF}^p(q) = e^{-q^2 l_B^{*2} c^2 / 4} \left[ L_p \left( \frac{q^2 l_B^{*2} c^2}{2} \right) - c^2 e^{-q^2 l_B^2 / 2 c^2} L_p \left( \frac{q^2 l_B^{*2}}{2 c^2} \right) \right],$$

has been absorbed into an effective CF interaction potential  $\tilde{v}(q) = v_0(q)[F_{CF}^p(q)]^2$ , for the case of the lowest electronic LL  $n = 0$ . The fact that one obtains a model with the same structure after restriction of the CF dynamics to a fixed CF-LL is precisely a mathematical manifestation of self-similarity. One simply has to take into account a renormalisation of the magnetic length and replace the original interaction potential by the effective CF potential. In analogy with the CF basis described above, one may find a representation in terms of C<sup>2</sup>Fs: after the introduction of a new pseudo-vortex density operator, one performs a variable change such as given by Eq. (5). These C<sup>2</sup>F-states may in principle be found at CF fillings  $\nu^* = p + \tilde{p}/(2\tilde{s}\tilde{p} + 1)$ , where each C<sup>2</sup>F carries  $2\tilde{s}$  flux quanta, and  $\tilde{p}$  denotes the number of completely filled C<sup>2</sup>F-LLs. In this approach, the 4/11 state corresponds to  $s = p = \tilde{s} = \tilde{p} = 1$ .

The iteration of this projection and the construction of such a new basis provides a possible framework for the description of higher generations of CFs. This approach yields a recursion formula for the filling factor  $\nu_j$  of the  $j$ -th generation of CFs (C<sup>j</sup>Fs),

$$\nu_j = p_j + \frac{\nu_{j+1}}{2s_{j+1}\nu_{j+1} + 1}, \quad (8)$$

where  $s_{j+1}$  denotes the number of flux pairs carried by the pseudo-vortex in the C<sup>j+1</sup>F, and  $p_j$  is the number of completely filled C<sup>j</sup>F-LLs. The IQHE of C<sup>j</sup>F is obtained for  $\nu_j = p_j$ , with integral  $p_j$ . Formally, the electronic case corresponds to  $j = 0$ , and thus  $\nu_0 = \nu$  and  $\nu_1 = \nu^*$ . Eq. (8) is a generalisation of the relation (1) between the electronic and the CF filling factors.

Note that the mathematical self-similarity on the level of the model structure does not *guarantee* a self-similarity between states. The existence of higher-generation CFs depends on the precise form of the CF interaction potential, which is *different* from the electronic one. Also the existence of first-generation CF states depends on the form of the effective interaction potential, which varies with the LL index although the structure of the model remains the same. Whereas in the two lowest LLs a 1/3 FQHE state has been observed, such a state has a higher energy than a two-electron bubble crystal in the second excited LL [18, 19]. The present model provides a framework for studying the competing phases in the case of C<sup>2</sup>Fs. Detailed energy calculations, performed directly in the thermodynamic limit, indicate that several C<sup>2</sup>F states are stable [20], in contrast to numerical-diagonalisation studies in Jain's wave-function approach. The latter method is restricted to the study of small systems and yields an alternation between compressible and incompressible states at  $\nu = 4/11$  as a function of the diagonalised number of CFs [21]. The extrapolation of these ambiguous results to the thermodynamic limit has misled the authors in the interpretation of this state [21].

At present, the only observed FQHE state, which may be interpreted in terms of spin-polarised C<sup>2</sup>Fs, is the 4/11 state [9]. Our formalism allows one to study various physical quantities of this state, such as its activation gap [11] or its energy with respect to competing CF-solid phases [20]. Here, we investigate the dispersion of its collective excitations in the single-mode approximation (SMA), which has first been applied to the Laughlin states in the lowest LL [14]. The excited state at  $\nu = 4/11$  may be obtained in the SMA by application of the projected CF-density operator  $\bar{\rho}(\mathbf{q})$  on the Laughlin state of CFs  $|\Omega\rangle$ ,  $|\psi_{\mathbf{q}}\rangle = \bar{\rho}(\mathbf{q})|\Omega\rangle$ . Its energy  $\Delta_{SMA}(q)$  may be expressed in terms of the projected static structure factor [22],

$$\bar{s}(q) = \frac{\langle \Omega | \bar{\rho}(-\mathbf{q}) \bar{\rho}(\mathbf{q}) | \Omega \rangle}{N_{el}} = (1 - \bar{\nu}^*) + 4\bar{\nu}^* \sum_{m=0}^{\infty} c_{2m+1} L_{2m+1} \left( q^2 l_B^{*2} \right) e^{-q^2 l_B^{*2} / 2},$$

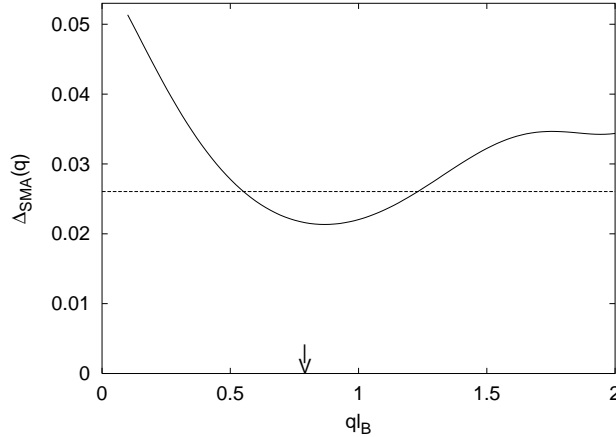


Fig. 3 – Dispersion of collective excitations at  $\nu = 4/11$ , calculated in the SMA. The arrow indicates the modulus of the reciprocal lattice vector of a CF Wigner crystal. The dashed line represents the value of the activation gap of the  $C^2F$  state, obtained in the Hamiltonian theory.

where  $\bar{\nu}^* = 1/3$  is the filling of the CF-LL  $p = 1$ , and the expansion coefficients  $c_{2m+1}$  characterise the Laughlin state  $|\Omega\rangle$ . The energy of the collective excitations,

$$\Delta_{SMA}(q) = 2 \sum_{\mathbf{k}} [\tilde{v}(|\mathbf{k} - \mathbf{q}|) - \tilde{v}(k)] \sin^2 \left( \frac{(k_x q_y - k_y q_x) l_B^{*2}}{2} \right) \frac{\bar{s}(k)}{\bar{s}(q)}, \quad (9)$$

is shown in Fig. 3. The position of the *roton-minimum*  $ql_B \simeq 0.85$  matches well the modulus of the reciprocal lattice vector of a CF Wigner crystal  $ql_B = \sqrt{\pi/3^{3/2}} \simeq 0.78$ , which one obtains from  $\bar{\nu}^* = 2\pi l_B^{*2}/A_{pc}$ , with the area of the primitive cell  $A_{pc}$  of a triangular lattice. In analogy with the interpretation of the magneto-roton of the electronic Laughlin states [14], one may view it as a tendency of the liquid state to crystallise. The dashed line in Fig. 3 represents the energy of the activation gap, calculated in the Hamiltonian theory [11]. Note that in contrast to Ref. [11], screening effects due to inter-CF-LL excitations, as well as finite-width effects, have been neglected here. In principle, the activation gap should coincide with the dispersion at large wave vectors, but it is known that the SMA yields less reliable results at large wave vectors [14]. However, the fact that the dispersion has a lower energy than the activation gap at the roton-minimum indicates that the SMA is accurate at intermediate values of  $q$ .

In conclusion, we have investigated the self-similarity of the quantum Hall effect in the framework of the Hamiltonian theory [13]. A change from the electronic to the CF basis - or more generally from  $C^{j-1}Fs$  to  $C^jFs$  - and the restriction of the particle dynamics to a fixed level yield the same model as the original one if one rescales the magnetic length and takes into account a modification of the interaction potential. Both the Hamiltonian [Eqs. (2) and (7)] and the commutation relations [Eqs. (3) and (6)] are reproduced in this projection scheme. However, it is not clear whether higher generations of CFs than  $C^2Fs$  are stable even in extremely pure samples; in this case, the CF-LL separation becomes extremely tiny, and the restriction to one level may therefore be a rather poor approximation because residual CF interactions are expected to lead to a strong level mixing. As an illustration of the relevance of the interacting-CF model, we have calculated the collective-excitation dispersion at  $\nu = 4/11$  in the SMA and compared the results to the  $C^2F$  activation gaps [11]. These results may shed some light on the similarities between electrons and (higher-generation) CFs, which appear to be essential for the understanding of QHE systems.

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